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The energy spectrum of cosmic ray positrons

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Abstract. A calculation has been made of the predicted energy spectrum of positrons generated by the positive pions and kaons produced in turn in the interactions of cosmic ray nuclei with the interstellar medium (ISM). A critical appraisal of the results and a comparison with the predictions of other workers are given. Experimental data on the measured energy spectrum have been considered and corrections have been made for the effect of solar modulation to enable derivation of the true interstellar spectrum of energetic positrons.

Comparison with prediction gives evidence for the main component of cosmic rays responsible for the positrons—protons—having passed through rather less matter in the ISM than is the case of heavier cosmic ray nuclei. Possible explanations are put forward.

1. Introduction

The energy spectrum of cosmic ray positrons arriving at the top of the earth's atmosphere can, in principle, provide useful information about the propagation of cosmic rays in the Galaxy. A number of authors have already examined the problem of interpreting the experimental data (e.g. Dilworth *et al* 1974, Badhwar *et al* 1975 and Orth and Buffington 1976; the last mentioned authors have given a useful summary of previous calculations); the justification for another examination is based on several factors: there are significant differences in previous predictions, most have not considered the important region below 1 GeV, the corrections to the experimental intensities to allow for solar modulation are not straightforward and need detailed analysis, and the interpretation of the discrepancy between the observed and expected spectra needs further consideration.

In what follows, a detailed examination is made of the interactions between cosmic ray nuclei and the interstellar medium (ISM) and the propagation of the resulting positrons is considered. The experimental intensities are examined and corrected so far as is possible for the modulation produced by the solar wind to give the presumed spectra in interstellar space. Comparison with expectation then allows the effective path length in the ISM to be derived as a function of energy. This determination of the cosmic ray 'grammage' is then compared with the same quantity derived from the spectrum of masses of primary nuclei which arise because of fragmentation of nuclei in their passage through the ISM and possible interpretations are considered.

A comparison is also made with the predictions for the positron 'grammage' of the closed Galaxy model of Rasmussen and Peters (1975) and Peters and Westergaard (1976).

2. Derivation of the expected positron spectrum

2.1. Sources of positrons

A variety of sources of positrons in the Galaxy spring to mind: secondaries in the $\pi^+ \rightarrow \mu^+ \rightarrow e^+$ chain (and from various kaon decay modes) produced by interactions of cosmic ray nuclei with the ISM, e^+ produced in individual sources, decay products of excited nuclei of both the ISM and cosmic rays and e^+e^- pairs from γ-ray interactions in the ISM. In the present work we confine attention to energies above 100 MeV and this means that the yield of β^+-active nuclei will be quite negligible (see Ramaty 1974 for a detailed discussion of positron sources). The γ-ray flux in the Galaxy is reasonably well known (see, for example, Dodds *et al* 1976 for a summary) as is the attenuation length for pair production in the ISM. Calculation shows that this source is negligible too except perhaps in very rare situations where generation takes place in very dense clouds. The situation regarding 'primary' positrons is not clear but, judging by the paucity of even anti-protons in the primary beam the ratio of anti-matter to matter amongst accelerated particles in our Galaxy is very small (an upper limit of $1{\cdot}4 \times 10^{-3}$ is quoted by Evenson 1972 for $Z = 2$ and energies in the range 0·2–4·3 GeV/nucleon) and there is thus no support for the possibility of a significant flux of 'primary' positrons.

It is concluded, therefore that the bulk of the cosmic ray positrons originate from the decay of secondaries produced in the interactions of cosmic rays with the ISM, principally from the $\pi^+ \rightarrow \mu^+ \rightarrow e^+$ sequence.

2.2. Energy range covered by the calculations

The positron energy range for which calculations have been made is 50 MeV–50 GeV. The lower limit has been chosen firstly to simplify the calculations of the positron energy distribution for a given π^+ energy and secondly there are big uncertainties in the experimental measurements below this limit so that no conclusions can be drawn from comparison. The upper limit also follows from reasons of poor experimental data on the positron flux; at energies about 20 GeV the statistical errors are at present very great indeed.

2.3. Primary cosmic ray spectrum

A prerequisite for the calculation of the positron spectrum is the magnitude of the spectra of the various nuclear components in interstellar space (the interstellar spectra, ISS). The ISS adopted for protons is:

$$J(E) = \begin{cases} 9{\cdot}35 \times 10^{3} E^{-2{\cdot}6}\ \mathrm{m^{-2}\,s^{-1}\,sr^{-1}\,GeV^{-1}} & \text{for } E < 5\ \mathrm{GeV} \\ 1{\cdot}74 \times 10^{4} E^{-2{\cdot}75}\ \mathrm{m^{-2}\,s^{-1}\,sr^{-1}\,GeV^{-1}} & \text{for } E > 20\ \mathrm{GeV} \end{cases}$$

where E is the total energy. A smooth transitional region is taken between 5 and 20 GeV.

The form below 10 GeV comes from the work of Comstock *et al* (1972) and there is some measure of justification in that Dodds *et al* (1976) have shown that, with a number of assumptions, the γ-ray flux from interactions in the ISM leads to this spectrum too. The question of the magnitude of this important spectrum is taken up again later.

At energies above 10 GeV the form given above comes from a recent survey by Juliusson (1975); it owes much in the important energy region to the measurements of Ryan *et al* (1972).

The ISS adopted for α-particles has been taken to be 11% and 4% of the proton intensity for the same energy per nucleon for the two energy ranges respectively with, again, a smooth transitional region. The fall in the α/p ratio with increasing energy is a prominent feature of the experimental observations.

It is assumed implicitly that the spectral shape of the cosmic ray nuclei adopted is maintained throughout the region of the Galaxy contributing to the measured positrons. It is most likely that there is a gradient of cosmic rays in the Galaxy, at least for the energies of concern here, and evidence has been put forward for such a gradient (Dodds *et al* 1976) so that the magnitude of the intensity is a function of position. However, for positrons of energy below about 10 GeV their propagation should be nearly the same as that of cosmic ray (CR) nuclei; furthermore since we can almost certainly neglect the energy losses of the positrons their distribution should follow that of the CR nuclei. Thus, only the CR spectral shape needs to remain the same as in the vicinity of the solar system. These points are considered again, in § 6.

2.4. *Composition of the interstellar medium*

The relevance of the composition of the ISM to the problem of cosmic ray interactions has been considered recently by Dodds *et al* (1976). Attention was drawn to the fact that the composition varies from place to place (see Trimble 1975 for a more detailed summary) and it is likely that it differs appreciably from average near to cosmic ray sources. In the present application, however, where the positron flux will be used to derive the mean mass of ISM traversed by cosmic rays in producing positrons and this will be compared with the mass traversed by cosmic ray nuclei as they fragment, then the actual composition is not critical. Dodds *et al* derive a value for $\bar{M}$, the effective mass per hydrogen nucleus, of 1·4; this value can be compared with the overall 'cosmic abundance' value of 1·36 quoted by Allen (1973). Here we adopt 11 hydrogen atoms per atom of helium (and negligibly smaller numbers of heavier nuclei), corresponding to $\bar{M} \simeq 1{\cdot}35$.

2.5. *Proton–proton and proton–helium interactions*

The experimental data for π^+ production have been taken for a wide energy range of proton kinetic energy, from near threshold to 200 GeV. Data for the lowest energies (<1 GeV) come mainly from Mescheryakov *et al* (1955) and Naganov *et al* (1957) who used counter techniques for detecting the positive pions. In the few GeV proton energy region the data have been taken from bubble chamber measurements by Bugg *et al* (1964), Fowler *et al* (1956), Fickinger *et al* (1962), Melissinos *et al* (1962), Coletti *et al* (1967) and Alexander *et al* (1967). Finally, for higher energies we have used the π^+ production inclusive cross sections given by Allaby *et al* (1970) and Whitmore (1974).

Integrating the π^+ energy spectra over the primary proton intensity gives the π^+ production rate from p–p interaction and this is presented in figure 1. The presence of α-particles in the cosmic radiation as well as helium in the interstellar matter gives only small correction to the π^+ production spectrum and it is sufficiently accurate to assume that the contribution from p–He interactions is proportional to $n_{\mathrm{pHe}}\sigma_{\mathrm{in}}$ i.e. the product of the total π^+ mean multiplicity over all inelastic channels and the inelastic cross

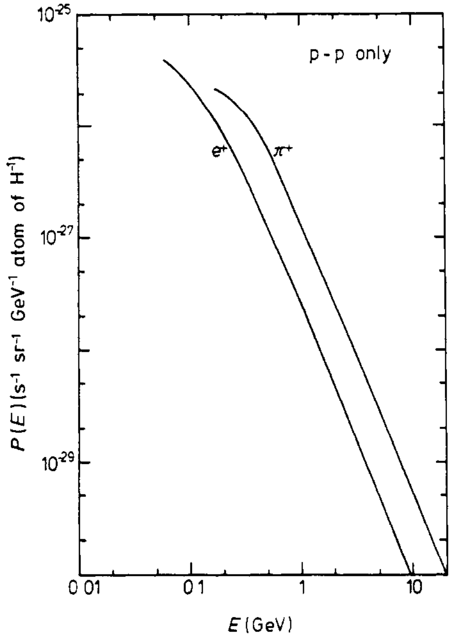

Figure 1. Production spectra of positive pions and positrons for cosmic ray proton–interstellar hydrogen collisions. The ordinate $P(E)$ is the yield per hydrogen atom $(\mathrm{s\,sr\,GeV})^{-1}$.

section. So, if M is defined as the factor by which the π^+ production rate per gram of hydrogen from p–p interactions only has to be multiplied in order to get the actual production rate taking place in the Galaxy, then

$$M=\left(1+\frac{n_{\mathrm{pHe}}\sigma_{\mathrm{pHe}}N_{\mathrm{He}}}{n_{\mathrm{pp}}\sigma_{\mathrm{pp}}N_{\mathrm{H}}}+\frac{n_{\alpha\mathrm{p}}\sigma_{\alpha\mathrm{p}}j_{\alpha}}{n_{\mathrm{pp}}\sigma_{\mathrm{pp}}j_{\mathrm{p}}}\right)\left(1+4\frac{N_{\mathrm{He}}}{N_{\mathrm{H}}}\right)^{-1}$$

where n denotes the average π^+ multiplicity; σ, the inelastic cross section; N, the number of interstellar gas atoms per unit volume; j, the cosmic ray intensity at the energy contributing most to the positron intensity.

As a matter of fact a distinction should be made between the multiplicity in proton–helium and α–hydrogen interactions, because only the multiplicity of comparatively high energy pions is relevant to our problem, and this is somewhat different in these two interactions. However, the distinction causes only a small change in M, so that we assume $n_{\mathrm{pHe}}=n_{\alpha\mathrm{p}}$, and thus

$$M=\left[1+\frac{(n\sigma)_{\mathrm{pHe}}}{(n\sigma)_{\mathrm{pp}}}\left(\frac{N_{\mathrm{He}}}{N_{\mathrm{H}}}+\frac{j_{\alpha}}{j_{\mathrm{p}}}\right)\right]\left(1+4\frac{N_{\mathrm{He}}}{N_{\mathrm{H}}}\right)^{-1}.$$

Table 1 represents the assumed characteristics for p–α interactions for different proton (nucleon in α-particle) kinetic energies and the value of the correction factor M.

The corresponding π^+ energy for which the above correction should be applied is a factor of 3–4 less than the proton energy.

The total number of positrons produced by cosmic ray protons has been calculated in a straightforward fashion to be $8{\cdot}6\times10^{-26}$ (s atom H)$^{-1}$ giving $5{\cdot}2\times10^{-2}$ (s g of H)$^{-1}$.

The median proton kinetic energy for the total e^+ flux is 1·5 GeV.

Table 1.

E_K (GeV)	$\frac{j_\alpha}{j_p}$	$\frac{n_{pHe}\sigma_{pHe}}{n_{pp}\sigma_{pp}}$	M
1	0·11	1·36†	0·94
10	0·11	4‡	1·3
100	0·04	4·5†	1·1

† Value adopted from measurements by Riddiford and Williams (1960).
‡ Value adopted from Hayakawa (1969).

2.6. *Derivation of the production rate of positrons from pions*

The positron energy spectrum for a positive pion of given energy, E_π, can be evaluated analytically if attention is restricted to the high energy region, where the approximation $\beta_\pi = \beta_\mu = \beta_e = 1$ is valid. We find

$$F_e(E)\,dE = \frac{2u}{u-1}\left\{\frac{2}{3}\left[1-\left(\frac{E}{E_\pi}\right)^3\right]-\frac{3}{2}\left[1-\left(\frac{E}{E_\pi}\right)^2\right]-\ln\frac{E}{E_\pi}\right\}\frac{dE}{E_\pi}$$

for $E_\pi/u < E < E_\pi$, and

$$F_e(E)\,dE = \frac{2u}{u-1}\left[\frac{2}{3}(u^3-1)\left(\frac{E}{E_\pi}\right)^3-\frac{3}{2}(u^2-1)\left(\frac{E}{E_\pi}\right)^2+\ln u\right]\frac{dE}{E_\pi}$$

for $E < E_\pi/u$ where $u = (m_\pi/m_\mu)^2 = 1{\cdot}75$. If in addition the π^+ spectrum has a power law form $A_\pi E_\pi^{-\gamma}$ then the resulting e^+ energy distribution will have the same slope:

$$F_e(E) = A_e E^{-\gamma}$$

where

$$A_e = \frac{12(1-u^{-\gamma})}{\gamma^2(\gamma+2)(\gamma+3)(1-u^{-1})}A_\pi.$$

This equation is the same as that derived by Orth and Buffington (1976) and it is important to note, following their remarks, that it gives a yield typically 25% smaller than results when the muon decay asymmetry is ignored (some earlier calculations of the positron intensity neglected the asymmetry).

The effective pion energy for a given positron energy is higher by a factor of about 3·7, assuming $\gamma = 2{\cdot}75$.

In the lower energy region, where β_π, $\beta_\mu \neq 1$, the integrals in the formula for the positron spectrum have a complicated form and must be solved numerically. The only limitation of our calculation is that it applies to positron energies bigger than $m_\mu c^2/2 = 52{\cdot}5$ MeV.

The result of the calculations is presented in figure 1, where the production spectra for π^+ and e^+ from p–p interactions only are given so that they can serve as a basis for calculations for different choices of interstellar matter composition, cosmic ray abundances and nuclear interaction characteristics. For positron energies bigger than a few GeV the spectrum approaches a limit of

$$P(E) = 3{\cdot}5\times10^{-28}E^{-2{\cdot}75}\ (\text{s sr GeV atom H})^{-1}$$

or $2{\cdot}7\times10^{-3}E^{-2{\cdot}75}$ (s GeV g of H)$^{-1}$.

2.7. Additional contribution from kaons

In addition to positrons from pions a small contribution arises from the various decay modes of neutral and charged kaons. Orth and Buffington (1976) have considered this aspect in some detail and they show that, for production of $e^{+}+e^{-}$, there is an enhancement factor $K(E)$ to apply to the electron spectrum given by

$$K(E) = 1{\cdot}10 + 0{\cdot}05 \lg(E) \qquad \text{with } E \text{ in GeV.}$$

Insofar as neutral kaons give a larger contribution than charged kaons the factor will be nearly the same for e^{+} and e^{-} and we adopt this form here for positrons alone (an approximate check has been made at 1 GeV where $K(E)$ is found to be within ~2% of Orth and Buffington's value).

Inclusion of kaons gives a total yield of positrons from p–p collisions:

$$P(E) = \begin{cases} 3{\cdot}9 \times 10^{-28} E^{-2{\cdot}73} \text{ (s sr GeV atom H)}^{-1} \\ 2{\cdot}9 \times 10^{-3} E^{-2{\cdot}73} \text{ (s GeV g of H)}^{-1} \end{cases} \qquad (\text{for } 5\text{ GeV} \leq E \leq 100\text{ GeV})$$

and yields higher by the factor M (table 1) for CR–ISM collisions.

3. Comparison of the calculated positron spectrum with those of other workers

A comparison is made in figure 2 of the calculated positron production spectra related to 1 g of interstellar material, i.e. allowing for both other nuclei besides protons in the cosmic ray beam and all the constituents of the ISM. It will be seen that there are significant differences in predictions amounting to as much as 35% in some cases. The majority of the differences arise from disparities in input data; in some cases we know that the later calculations are more reliable because they utilize new accelerator results which were not available hitherto and significant effects disregarded earlier. We regard the residual uncertainty in our own prediction to be about ±15% at all energies.

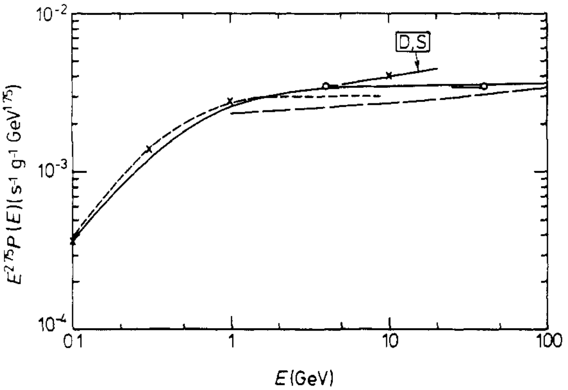

Figure 2. The production spectrum of positrons per second per gram of ISM. O——O, Badhwar *et al* (1975); — —, Orth and Buffington (1976); - - - - -, Ramaty (1974); D, S, and ×, Daniel and Stephens (1975); ——, present work.

4. Comparison with experimental measurements

4.1. Correction for solar modulation

Measurements of the positron flux extended downwards to 10 MeV or so but the corrections for solar modulation become increasingly inaccurate below 100 MeV so that attention in the present work is confined to energies above 200 MeV where corrections may be made with some degree of confidence. The object is to take experimental results on the positron intensity made at different times and correct them in turn so as to correspond to the ISS.

A number of authors have discussed the modulation process in some detail and it is true to say that there is, as yet, no general concensus as to the details. However, the treatments appear to be sufficiently alike for the present purpose, as will be demonstrated later.

Kolomeets and Zusmanovich (1971) have developed a procedure and Aitmuhambetov *et al* (1975) have applied this to the modulation of the electron component. These authors show that for energies greater than 0·3 GeV the factor by which the electron intensity decreases can be well described by the function $\exp(K(t)/E)$ where $K(t)$ is a time-dependent function and E is the energy. Aitmuhambetov *et al* give values of $K(t)$ for different years up to 1972 (figure 3) and we have adopted their values to demodulate

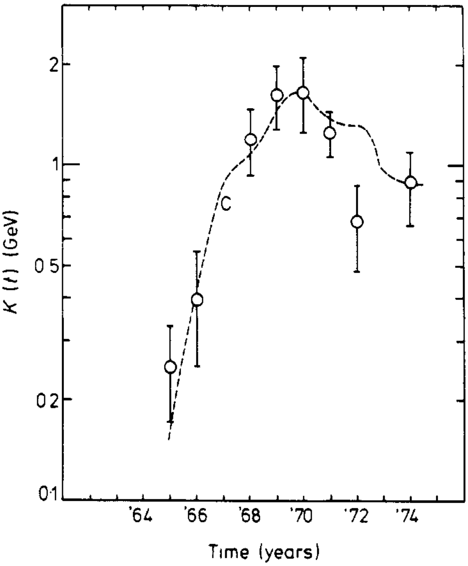

Figure 3. The demodulation factor $K(t)$ from the analysis of Aitmuhambetov *et al* (1975) (open circles) and the modulation parameter of Charakchyan *et al* (1975) denoted C. The two are normalized for 1970. The point for 1974 has been evaluated by the present authors.

the observations of Fanselow *et al* (1969) and Daugherty *et al* (1975). In order to obtain a correction for the measurements of Hartman and Pellerin (1976), which were obtained in 1974, measurements on the e^- spectrum taken by these authors were used in conjunction with a similar experiment made in 1972 (Daugherty *et al* 1975) together with measurements on e^- made by the Chicago group (Meyer *et al* 1971, Fulks and Meyer 1973 and Caldwell *et al* 1975). The value of K which fits the observations best above 300 MeV is 0·88 ± 0·22 and this has been used to demodulate the 1974 positron flux. Table 2 summarizes the modulation factors used in the present work and figure 3 gives $K(t)$ together with estimated errors. It will be noticed that K (1974) derived in the way described does not fall as rapidly after 1970 as might have been expected from reasons of symmetry. This arises essentially because the measurements by the Chicago group (see for example the summary of Caldwell *et al* 1975) show very little variation in the electron intensity (in the range 600–1250 MeV) from 1972 to 1974.

Table 2. Modulation factors applied to the data.

Authors	Measurement period	Modulation factor K (GeV)
Fanselow *et al* (1969)	1965–66	0·31 ± 0·12
Daugherty *et al* (1975)	1972	0·68 ± 0·20
Buffington *et al* (1975)	1972–73	0·7 ± 0·2
Hartman and Pellerin (1976)	1974	0·88 ± 0·22

The important problem of modulation has also been considered in some detail using other approaches. Charakchyan *et al* (1975) were able to fit stratospheric ion chamber data at different rigidity cut-offs using a modulation parameter proportional to $S^{0\cdot8}\lambda^{-1\cdot2}$ where S is the sunspot number and λ their mean helio latitude. We have derived this parameter for the period 1965–74 using the Zurich sunspot numbers (Ahluwalia 1975) and values of λ from the work of Mendell and Korff (1975) with the result shown in figure 3, (the parameter is normalized to K (1970)). It is reassuring to see that it follows roughly the adopted form of $K(t)$ and, in particular, predicts only a small reduction from 1970 to 1974.

It is relevant to note that the unusual behaviour after 1970 has been noted by many workers and may well be related to the solar field reversal in 1969 (Cini-Castagnoli *et al* 1975).

The modulation effect is examined still further in figure 4 which shows the ratio of spectral intensities inferred for the ISS to those in 1965 (sunspot minimum). The caption indicates the source of the information.

Some comments are necessary about the disparity in the results. Curve A is probably high because of the point made by Aitmuhambetov *et al* that the ISS electron derived from radioemission data is probably an overestimate. The curve for protons is derived in an indirect manner but is probably close to the truth; however, it is not directly comparable with that for electrons because of the effect of energy losses in the interplanetary medium which affect electrons and protons differently.

The corrected positron intensities indicated in figure 5 have error bars which make allowance for uncertainties in the modulation correction as well as inherent statistical errors. Despite this fact, there is seen to be a rather large spread in intensities, somewhat greater than might have been expected from purely statistical errors alone. However, there is some measure of confidence in the modulation corrections in that for the lowest energy range, below 1 GeV, where there is the possibility of comparing different sets of data, and the 1972 and 1974 intensities alternate, there seems to be no systematic trend of intensity with date. It is also relevant to point out that there are certainly some

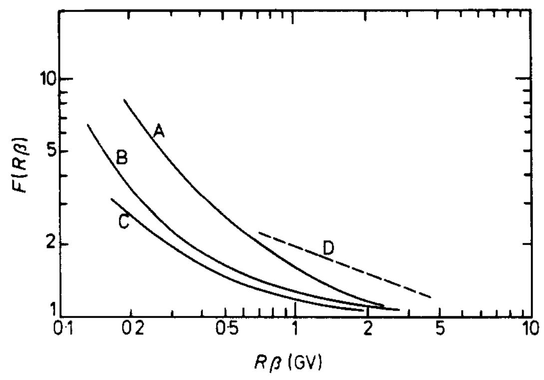

Figure 4. Ratio of intensity of the interstellar spectrum to that at solar minimum (1965) versus product of rigidity R and velocity β. The full curves refer to electrons and the broken curve to protons. Curve A: $F(R\beta)$ derived from a comparison of ISS of electrons derived from radio emission data by Bulanov and Dogel (1974) and the analysis of measured spectra by Aitmuhambetov *et al* (1975). The spectra were normallized at 10 GeV. Curve B: $F(R\beta)$ using $K = 0{\cdot}25$ GeV (Aitmuhambetov *et al* 1975). The variation adopted here. Curve C: $F(R\beta)$ derived from a comparison of the 1965 electron spectrum measured by Webber *et al* 1(1973) with that derived from radio emission data by Cummings *et al* (1973); normalization above 4 GeV. Curve D: protons, analysis by Dodds *et al* (1976).

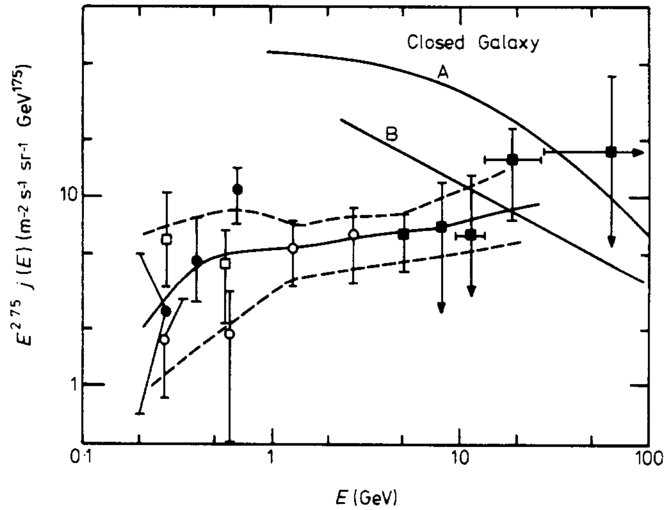

Figure 5. The measured interstellar positron spectrum (corrected for modulation by us). The closed Galaxy predictions relate to: curve A, the model of Rasmussen and Peters (1975) with an assumed mean ISM density of 1 atm cm^{-3} (after French and Osborne 1976a, b); and curve B, the model of Peters and Westergaard (1976) (small corrections have been applied for bremsstrahlung loss). Experimental data: ■, Buffington *et al* (1975); ○, Fanselow *et al* (1969); □, Daugherty *et al* 1(975); ●, Hartman *et al* (1975). The middle curve is the best estimate and the broken curves are approximate one standard deviation limits.

errors in the positron intensities which are not related to the modulation corrections. For example, the e^+/e^- ratio at a particular energy should surely be constant; however the ratios found in two similar experiments by Daugherty *et al* (1975) and Hartman and Pellerin (1976) differ quite significantly.

Despite the errors we have drawn what is considered to be the best estimate of the interstellar positron intensity and this is shown in figure 5 along with estimated one standard deviation upper and lower limits.

4.2. Comparison with predicted spectra

In first approximation, the predicted positron spectrum follows by multiplying the production rates of figure 2 by the assumed path length, λ, and dividing by 4π (to give the intensity rather than the omnidirectional flux). A value of $\lambda = 6{\cdot}5\ \mathrm{g\ cm^{-2}}$ has been adopted as a datum and the corresponding spectrum is shown as $T \rightarrow 0$ in curve A of figure 6.

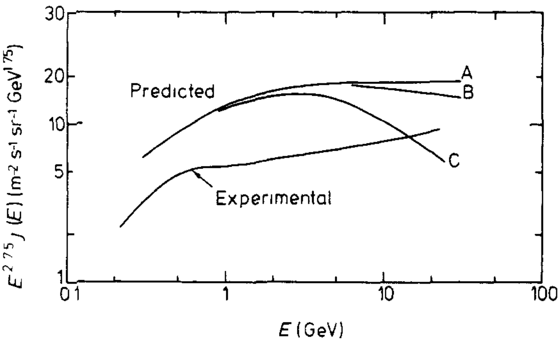

Figure 6. Comparison of the best estimate of the observed positron spectrum (marked 'experimental') with that predicted for a mean path length of $7{\cdot}4\ \mathrm{g\ cm^{-2}}$ in the ISM and the alternative mean lifetimes in the Galaxy indicated. Curve A, $T \rightarrow 0$; curve B, $T = 3\times10^6$ yr; curve C, $T = 2\times10^7$ yr. (Curves B and C are approximate.)

In fact, allowance must be made for energy loss against photons (ICE) and in the synchrotron and bremsstrahlung processes. The procedure for this is known and is outlined very briefly in the appendix. With an adopted photon energy density of $0{\cdot}7\ \mathrm{eV\ cm^{-3}}$ (starlight plus $2{\cdot}7K$ radiation) and an effective mean magnetic field of 4 μ gauss application of the attenuation factors for two different lifetimes gives the spectra indicated in figure 6; the predicted curves now relate to $\lambda \simeq 7{\cdot}4\ \mathrm{g\ cm^{-2}}$.

$T = 3\times10^6$ years represents the conventional mean life for cosmic rays and $T = 2\times10^7$ yr is the most probable lifetime derived recently from the $\mathrm{Be}^{10}/\mathrm{B}$ ratio measurements of Garcia-Munoz *et al* (1975).

4.3. Derivation of mean path length

The mean path length from the positron data is found simply from the data of figure 5 and the result is shown in figure 7 as smooth curves for the different mean lifetimes. The limits to the spectrum of figure 5 and the mean intensities give the results for λ shown in figure 8 for the conventional lifetime ($T = 3\times10^6$ yr).

It will be seen that over the range of positron energy $1 < E(\mathrm{e}^+) < 10$ GeV, $\langle\lambda\rangle = 2{\cdot}8 \pm 0{\cdot}8\ \mathrm{g\ cm^{-2}}$ with perhaps slightly higher values outside these energy limits.

A comparison will be made with the mean free path derived from measurements of isotropic abundances later.

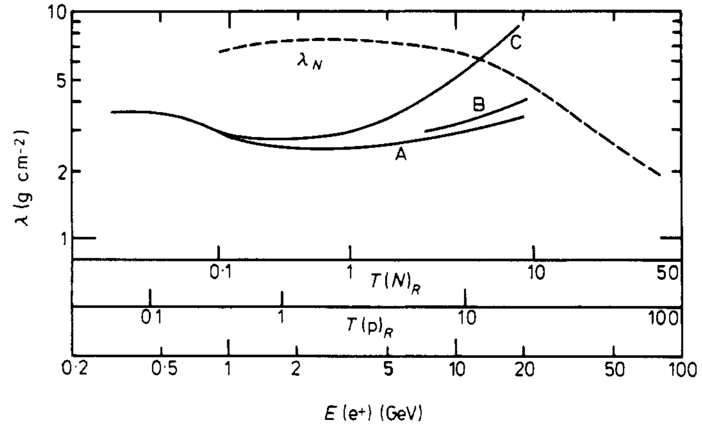

Figure 7. The derived mean path length in the ISM, λ, from the positron measurements (for $T \to 0$, 3×10^6 yr and 2×10^7 yr). The dotted line is an estimate, from the data given in figure 8, of the dependence of the mean free path for nuclei, λ_n, from the interpretation of various measurements of isotopic composition. These relate to energy per nucleon and the scales have been chosen so as to refer to particles (including positrons) of the same rigidity: $T(N)_R$ relates to kinetic energy per nucleon of the same rigidity as positrons of energy E and $T(\text{p})_R$ relates to protons of the same rigidity. Curve A, $T \to 0$; curve B, $T = 3 \times 10^6$ yr; curve C, $T = 2 \times 10^7$ yr. (Curves B and C are approximate.)

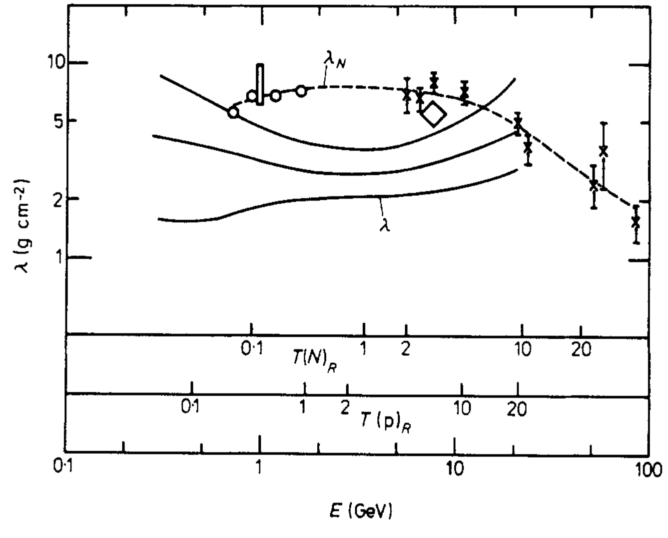

Figure 8. Predicted dependence of λ on energy from positron data (with one standard deviation limits and $T = 3 \times 10^6$ yr). The experimental values relate to interpretations of the isotopic composition: ○, from the summary of ^{3}He/^{4}He ratios collected by Shapiro and Silberberg (1974), grouped by us and scaled up by a factor 1·2 as described in the text; ▯, range quoted by Meyer (1975); ⨯, summary by Juliusson (1975); ◇, Shapiro *et al* (1975).

5. Relevance of the results to the closed Galaxy model

Recently Rasmussen and Peters (1975) have proposed a closed Galaxy model for cosmic ray confinement. The expected positron flux in this model is difficult to reconcile with experiment (French and Osborne 1976a) unless very large cosmic ray modulation is assumed. Such an assumption would lead to many other problems (figure 4 indicates that the demodulation increase is most unlikely to be large enough above 1 GeV). Badhwar and Stephens (1976) have also made objections to the closed Galaxy model on the basis of both electron and radio emission data.

The prediction is shown in figure 5 (curve A: closed Galaxy I) for the conditions indicated in the caption. Next Peters and Westergaard (1976) proposed a second version of the closed Galaxy model and we have calculated the intensity predicted by this model. In the model the positron flux should have two components; the 'old' and the 'young'. The old component is that of positrons produced in a large containment volume in which the mass of interstellar gas is about 4×10^{10} solar masses. As there is no leakage from this volume, the positron flux is determined by energy losses and production rate only. The young component is that of positrons produced in spiral arms from which they escape after traversing a mean grammage λ depending on rigidity as $\lambda = \lambda_0 R^{-0\cdot 5}$, for $R > 2$GV. The mass of interstellar gas in the spiral arms is assumed to be about 100 times less than the mass of the whole containment volume and, assuming that the radius of this volume is 15 kpc, this gives a density of about 0·1 hydrogen atoms cm^{-3}.

To evaluate the old positron flux the calculations of Ramaty and Westergaard (1976) (their curve III in figure 2) have been used assuming that the cosmic ray flux for energies of interest in the large volume is 20% of that in the spiral arms. The adopted magnitude of the magnetic field is 4 μG.

The young positron flux is determined by the grammage given above. The value of λ_0, which fits the abundances of nuclei, depends on their path length distribution, but for the positron case only the mean value is relevant. In order to make a better fit to the experimental data we have chosen the smaller value $\lambda_0 = 15 \text{ g cm}^{-2}$ from the two adopted in the model. The resulting positron intensity is presented in figure 5. It is essentially that of young positrons, the maximum contribution of the old particles being only about 12%. Thus, the second version of the closed Galaxy model is, from the positron point of view, practically equivalent to the leaky box model with a rather strongly energy-dependent grammage. We see that it is not possible to reconcile the predicted curve with experimental data.

6. Discussion

A comparison of our calculated grammage from the positron calculations with that obtained from nuclei abundances λ_N is presented in figures 7 and 8. Figure 7 shows the best line through the experimental determinations and the points themselves are given in figure 8.

The values have been derived from measurements on a variety of isotopes. Those indicated ^{3}He/^{4}He come from a summary by Shapiro and Silberberg (1974); the original values related to fragmentation in hydrogen and we have multiplied by 1·2 to give the path lengths relevant to interactions in the ISM (the interstellar helium and more massive atoms are less efficient in fragmenting nuclei than are hydrogen nuclei for the same grammage; we are grateful to Professor R Silberberg for discussions on this point). Shapiro and Silberberg (1974) have also used the ratio of fluxes of L and M nuclei (L/M ratio) to determine λ for rigidities above 4 GV/nucleon. More recently (Shapiro *et al* 1975) the authors have used new data in both primary cosmic rays and on fragmentation cross sections to derive $\lambda = 5{\cdot}5\ \text{g cm}^{-2}$ ($\pm 15\%$) near 3 GeV/nucleon and this is the value shown in figure 8. It is necessary to note that the values below one or two GeV/nucleon are somewhat uncertain because of the different effect of modulation on the various nuclear species.

The values of λ and λ_N, must be compared for the same rigidity rather than the same energy. The rigidity of a nucleon in a nucleus is twice that of a proton for the same energy and so 1 GeV positrons correspond to nuclei at around 125 MeV kinetic energy per nucleon, whereas for the highest energies the positron energy will be twice that of the nucleon.

Inspection of figures 7 and 8 shows a discrepancy in the region 1–10 GeV positron energy, whereas for 20 GeV there is seen to be reasonable agreement, although it must be said that there is a downward trend of λ_N with energy which is opposite to that of λ.

At this stage comparison can be made with the conclusions of other workers. Dilworth *et al* (1974) come to a very similar conclusion to ourselves; Orth and Buffington find $\lambda \simeq 4\ \text{g cm}^{-2}$, a result that follows from their lower predicted production rate, but they also draw attention to the inconsistency in the trend with energy of λ_N and λ. Badhwar *et al* (1975) find $\lambda = 4{\cdot}7 \pm 1{\cdot}5\ \text{g cm}^{-2}$ for $E > 4$ GeV despite calculating virtually identical production rates to our own: the discrepancy occurs because of the use of earlier data of Buffington *et al* (1974) which gave significantly higher intensities.

If our contention that λ is appreciably less than λ_N in the range 1–10 GeV is accepted then the simple model of the Galaxy with a homogeneous flux of cosmic rays and positrons and a rigidity-dependent leakage time is not valid. It also shows, by virtue of there being fewer positrons than expected, that it is most unlikely that a significant number of positrons are accelerated in the Galaxy, either in the sources themselves or as secondaries in ISM. Thus acceleration models with well separated origin and acceleration regions are not favoured. More sophisticated hypotheses are required.

A number of possibilities spring to mind. The first is that some of the nuclei seen as primary cosmic rays (in the energy range in question) are extragalactic in origin. In this case, perhaps half the grammage experienced by the nuclei occurs in remote extragalactic objects—the resulting positrons being lost by interactions with the 2·7K radiation. Whilst not impossible such a situation is unlikely because of the need to postulate nearly equal contributions of Galactic and extragalactic nuclei. In what follows we assume implicitly that the nuclei (and the positrons) are of Galactic origin.

Another possibility is that low energy cosmic rays are kept in dense regions, perhaps near the sources, for considerable periods and half of the positrons do not emerge. This would depress the positron flux at low energies as seems to be the case. If the energy loss for positrons were by way of synchrotron losses then the nuclei would emerge unscathed (except for penetration of about $2\ \text{g cm}^{-2}$ material on average)—a necessary condition. A consequence is that the same sort of losses would be felt by e^- particles which, in a conservative model would be generated in the same sources as the nuclei. The drawback is that there should be considerable synchrotron radiation from these slowing-down electrons. An order of magnitude calculation has been made for the most likely sources—supernova remnants—and this shows that the emission probably exceeds the observed radio intensities (more work is necessary on this topic).

Other authors (e.g. Cowsik and Wilson 1973) have also put forward models in which nuclei traverse some of their 'grammage' near their sources and the rest elsewhere. If the particles are trapped near the source, or indeed in any volume to which the solar system is exterior, with an energy-dependent lifetime than an equality of λ and λ_N would not be expected. This is because the proton spectrum inside the source (volume) will be steeper than that locally and the positron flux will be lower than calculated above (Orth and Buffington 1976 have also made this point). This brings us back to the point made in § 2.3 about variations in the spectral shape over the Galaxy. A model having a somewhat similar philosophy has been put forward by Dilworth *et al* (1974) in which high energy particles spend less time in high density regions.

Osborne (1976, private communication) has drawn attention to the possibility that the source-trapping region could even be the galactic arms. There is the well known observation that the measured radio synchrotron flux is greater than expected on the basis of electrons with their locally measured spectrum spiralling magnetic fields of the local magnitude. French and Osborne (1976b) put forward an interpretation in terms of the solar system being in an inter-arm region where the synchrotron emissivity is less than that in the nearby spiral arms. There could, thus, be similar positron concentration in the arms.

Another explanation is that the cosmic ray nuclei seen at the earth have somewhat different sources: the protons being generated by objects which are, on average, rather closer than those giving rise to heavier nuclei. Such a model is reasonable insofar as protons of the energies in question (less than some tens of GeV) might be expected to originate in more common sources than heavier nuclei. A consequence would presumably be that the ratio of 'heavy' nuclei to protons would vary over the Galaxy. Some variation would be allowable by the only other relevant experimental data—that of γ-rays—but it seems unlikely that the value at the earth could differ by more than a factor of 3 from the local average (over say 2 kpc of the earth).

Taken on balance, the differential trapping model seems to be more likely.

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Appendix. Evaluation of the reduction of the positron intensity due to energy losses

A.1. Synchrotron ICE losses

In a leaky box model the equilibrium differential density of positrons $n(E)$ is described by the equation

$$q(E)=\frac{n(E)}{T}+\frac{\partial}{\partial E}(\dot{E}n(E)) \tag{A.1}$$

where $q(E)$ is the production rate and T the mean lifetime for escape.

If we assume that the energy losses have the form $\dot{E}=-bE^2$ (as is the case for the synchrotron radiation and the inverse Compton effect processes), and if the production rate has a power law energy dependence, $q(E)=q_0E^{-\gamma}$, then the solution to equation (A.1) is

$$n(E)=q_0E^{-\gamma}Tf_\gamma(bTE) \tag{A.2}$$

where

$$f_\gamma(bTE)=(bTE)^{\gamma-2}\exp\left(-\frac{1}{bTE}\right)\int_0^{1/bTE} y^{\gamma-2}\,\mathrm{e}^y\,\mathrm{d}y.$$

The function of $f_\gamma(bTE)$ describes the reduction of the positron density due to energy losses. The integral can be solved analytically for integral values of γ:

$$f_2 = 1 - \exp\left(-\frac{1}{bTE}\right) \qquad f_3 = 1 - bTEf_2 \qquad f_4 = 1 - 2bTEf_3.$$

The reduction factor for non-integral γ values is easily found by interpolation. Figure 9 represents the reduction factor as a function of γ for different values of the product bTE. Figure 10 gives the factor as a function of bTE for $\gamma = 2{\cdot}75$.

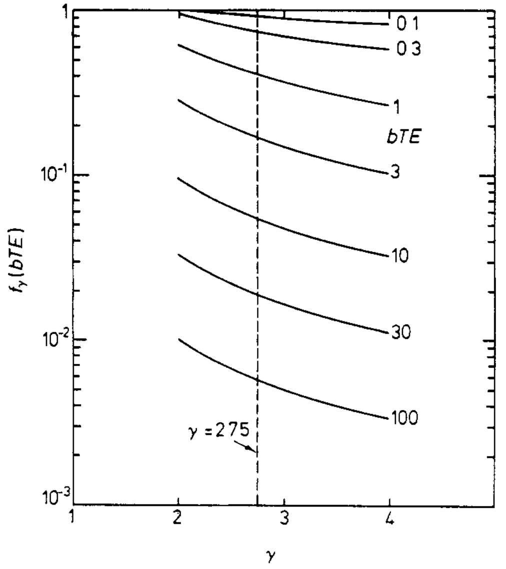

Figure 9. Reduction factor to be applied to the predicted positron intensity, $f_\gamma(bTE)$, to allow for E^2 losses. γ is the exponent of the differential production spectrum. The factors are given for different values of bTE. If $b = 1{\cdot}12 \times 10^{-16}$ (GeV s)$^{-1}$ then $bTE = 3{\cdot}53 \times 10^{-3}$ TE, where T is the mean lifetime in millions years and E is the positron energy in GeV.

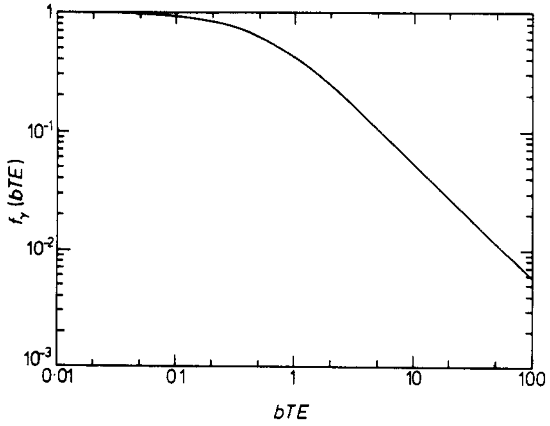

Figure 10. Reduction factor to be applied to the predicted positron intensity, $f_\gamma(bTE)$, to allow for E^2 losses. The quantities are defined in the caption to figure 9.

A.2. Bremsstrahlung losses

The small correction necessary for bremsstrahlung loss is best considered separately. In the low energy region where the E^2 losses are negligible we have $\dot{E}=-aE$ and the effect is equivalent to a decrease of the escape lifetime, T, so that the new lifetime T' satisfies the relation

$$\frac{1}{T'}=\frac{1}{T}+(\gamma-1)a. \tag{A.3}$$

The solution of equation (A.1) follows as $n(E)=q(E)T'$. The grammage for positrons, defined as the product of the average density of matter through which the positrons travel and their escape lifetime multiplied by their velocity, is then

$$\lambda=\lambda_a[1-(\gamma-1)\lambda_a/x_0]^{-1} \tag{A.4}$$

where x_0 is the radiation length of the ISM and λ_a is the grammage derived under the assumption of no bremsstrahlung loss.

The correction is a maximum a little above 10 GeV; at higher energies the E^2 losses dominate and the correction becomes smaller.

For $\gamma=2{\cdot}75$, $x_0\simeq 65\ \mathrm{g\ cm^{-2}}$ and $\lambda\sim 3\ \mathrm{g\ cm^{-2}}$ the upward correction to λ is: 6% at 1 GeV, 9% at 10 GeV and ~7% at 100 GeV; all for $T=3\times10^6$ years.

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